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MULTISENSOR CALIBRATION PROCEDURE FOR AN ALL WEATHER
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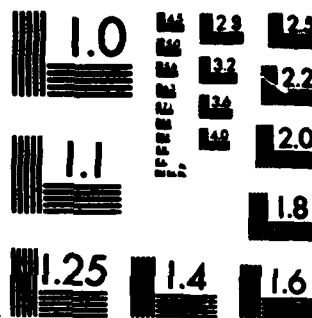
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TECHNICAL REPORT RG-83-17

**MULTISENSOR CALIBRATION PROCEDURE
FOR AN ALL WEATHER SHORT RANGE AIR DEFENSE
SYSTEM CONCEPT**

Wayne L. McCowan and Vicki C. Lefevre
Guidance and Control Directorate
US Army Missile Laboratory

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U.S. ARMY MISSILE COMMAND

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I. INTRODUCTION

An All Weather Short Range Air Defense System (A/W SHORADS) concept, which utilizes data from an on-board strapdown Inertial Measurement Unit (IMU) and target state updates from the ground to provide a midcourse guidance phase, is currently being studied. The missile rates and accelerations are measured by two low-cost multisensors contained in the IMU. In order to reduce the navigation errors which accumulate from multisensor error sources during flight, a pre-launch multisensor calibration is desirable.

Rockwell International [1] performed a study to develop a calibration scheme for use with multisensors on a fiber optics guidance missile (FOG-M) concept. This scheme, however, was developed for a vertical missile orientation at calibration and was not directly applicable to the A/W SHORADS case. The recommended calibration procedure from Reference 1 was, therefore, modified to perform for a level, instead of a vertical, missile calibration orientation and to be in accordance with the A/W SHORADS axis definitions. The resulting proposed calibration scheme is described in this report. It relies heavily on Reference 1 for nomenclature and for the framework within which this procedure was developed.

Section II presents the multisensor configuration for navigation and the calibration reference axes, along with the parameters to be calibrated. Section III presents the accelerometer calibration measurement equations and the iterative procedure used to calibrate the accelerometer parameters. Section IV presents the gyro calibration equations.

The proposed calibration procedure was programmed into an alignment subroutine in the six degree-of-freedom (6-DOF) A/W SHORADS digital simulation in order to examine its performance. The Appendix contains a description of the sequencing of operations for the calibration along with a listing of the calibration section and data outputs for several check cases.

II. SYSTEM CONFIGURATION

For the A/W SHORADS case, as in Reference 1, each multisensor is mounted with a calibration rotation axis \bar{R}_1 nominally perpendicular to the multisensor spin axis \bar{S}_1 . For the A/W SHORADS case, however, the multisensors are mounted into the missile in the orientation shown in Figure 1. This is the assumed navigation orientation, with $\theta_{R1} = -90^\circ$ and $\theta_{R2} = 0^\circ$ and provides a redundant pitch axis instead of a redundant yaw axis as is the case for FOG-M.

In order to perform the calibration, measurements are made by each multisensor at three positions: $\theta_{R1} = 0^\circ, -90^\circ, -180^\circ$. In addition, measurements are made by each multisensor as it is rotated between the 0° and -180° positions. The measurements made at the stationary orientations are used in calibrating the accelerometers and for some of the gyro calibrations. The data measured during the 180° rotation is used in the gyro calibration process. The parameters which are to be calibrated are shown in Table 1 (see Reference 1).

TABLE 1. PARAMETERS TO BE CALIBRATED

PARAMETER	DEFINITION
K_{A1}, K_{A2}	ACCELEROMETER SCALE FACTOR
δ_{A1}, δ_{A2}	ACCELEROMETER MISALIGNMENT ABOUT THE SPIN AXIS
$B_{A1}, B_{B1}, B_{A2}, B_{B2}$	ACCELEROMETER BIASES
E_1, E_2	ANGLE BETWEEN \bar{R} AND \bar{S} IS $(\pi/2-E)$ RADIANS
K_{G1}, K_{G2}	GYRO SCALE FACTOR
δ_{G1}, δ_{G2}	GYRO MISALIGNMENT ABOUT SPIN AXIS
$D_{A1}, D_{B1}, D_{A2}, D_{B2}$	GYRO DRIFT BIASES
D_{A1GSA}	ON-AXIS G-SENSITIVE DRIFT
D_{A1GSB}	CROSS-AXIS G-SENSITIVE DRIFT

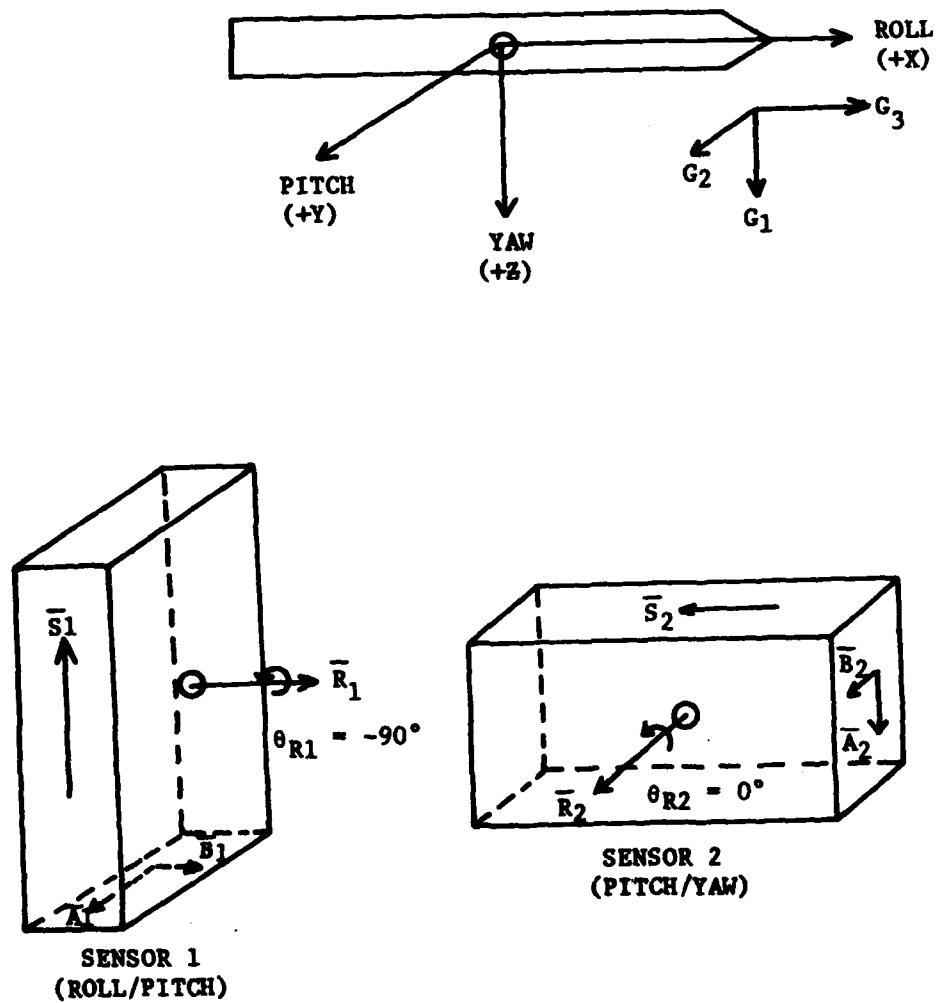


Figure 1, Multisensor orientation for navigation.

III. ACCELEROMETER CALIBRATION

A. Introduction

This section develops the equations used in the accelerometer calibration process for the A/W SHORADS concept. The orientation of the multisensor axes at each of the stationary measurement positions will be shown relative to a reference axis set defined as follows. Let multisensor 2 be at its navigation orientation, $\theta_{R2} = 0^\circ$. The reference roll, pitch and yaw axes for multisensor 2 are then given as

$$\text{ROLL}_2 = -\bar{S}_2 \quad (1)$$

$$\text{PITCH}_2 = \bar{B}_2 = \bar{S}_2 \times \bar{A}_2 \quad (2)$$

$$\text{YAW}_2 = \bar{A}_2 = \bar{R}_2 \times \bar{S}_2, \quad (3)$$

and are shown in Figure 2. Let multisensor 1 be at its navigation orientation, $\theta_{R1} = -90^\circ$. The reference roll, pitch and yaw axes for multisensor 1 are then given as

$$\text{ROLL}_1 = \bar{B}_1 = \bar{S}_1 \times \bar{A}_1 \quad (4)$$

$$\text{PITCH}_1 = \bar{A}_1 = \bar{R}_1 \times \bar{S}_1 \quad (5)$$

$$\text{YAW}_1 = -\bar{S}_1, \quad (6)$$

and are shown in Figure 3.

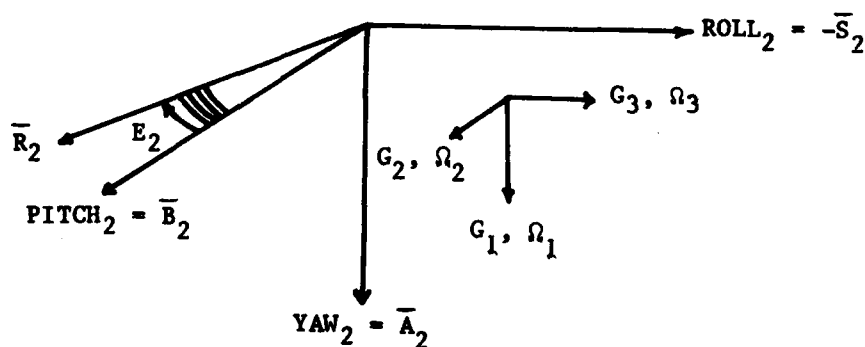


Figure 2. Reference axis set and multisensor 2 axes at $\theta_{R2} = 0^\circ$.

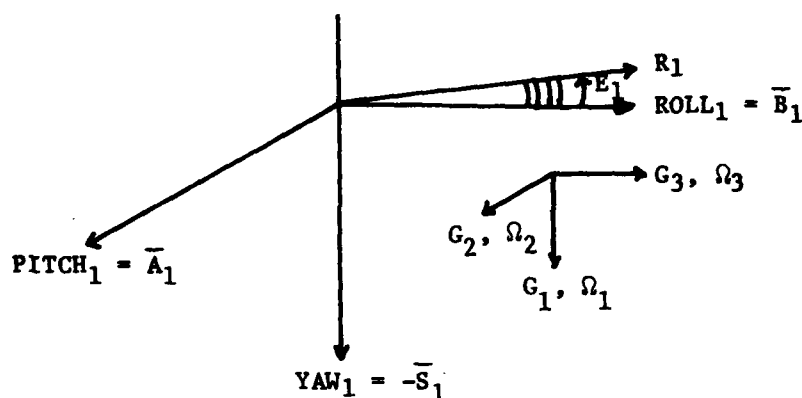


Figure 3. Reference axis set and multisensor 1 axes at $\theta_{R1} = -90^\circ$.

For multisensor 2, the $\theta_{R2} = 0^\circ$ orientation is taken to be a perfect orientation. Due to mechanical imperfections, the multisensor is not rotated by exactly 90° from one calibration position to another. This effect is modeled by $\theta_{R2} = -90^\circ + \beta_{24} \approx -90^\circ$ and $\theta_{R2} = -180^\circ + \beta_{22} \approx -180^\circ$. Another mechanical imperfection which is considered is the non-orthogonality of the calibration rotation and the spin axes. The actual angle between the two axes is taken to be $(\pi/2 - E_2)$ radians. For multisensor 1, the $\theta_{R1} = -90^\circ$ orientation is taken to be the reference point and rotation to $\theta_{R1} = \beta_{11} \approx 0^\circ$ and to $\theta_{R1} = -180^\circ + \beta_{12} \approx -180^\circ$ define the error angles β_{11} and β_{12} . The angle between \bar{R}_1 and \bar{S}_1 is $(\pi/2 - E_1)$ radians. These misalignment angles thus define the orientation of the actual sensor axes with respect to the reference axis set.

B. Multisensor 2 Accelerometer Calibration

Multisensor 2 is a pitch/yaw sensor. Its orientation during navigation is with $\theta_{R2} = 0^\circ$ and an ideal (reference) axis set for use in calibration is defined at this orientation as shown in Figure 2. Since the accelerometer axes are assumed to be misaligned about the spin axis by an angle δ_{A2} , the actual multisensor axes are located with respect to the reference axes as shown in Figure 4, where δ_{A2} is positive for a positive rotation about \bar{S}_2 .

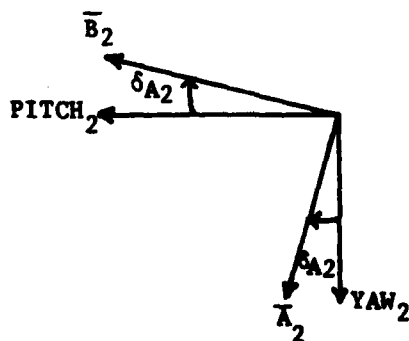


Figure 4. Accelerometer misalignment with respect to spin axis, $\theta_{R2} = 0^\circ$.

If one writes the equations for measurements along \bar{A}_2 and \bar{B}_2 , and applies the small angle approximation, the results are:

$$\bar{A}_{21} = K_{A2} G_1 \cos \delta_{A2} + G_2 \sin \delta_{A2} + B_{A2} \approx K_{A2} G_1 + \delta_{A2} G_2 + B_{A2} \quad (7)$$

$$\bar{B}_{21} = K_{A2} G_2 \cos \delta_{A2} - G_1 \sin \delta_{A2} + B_{B2} \approx K_{A2} G_2 - \delta_{A2} G_1 + B_{B2}. \quad (8)$$

When multisensor 2 is rotated to $\theta_{R2} = -90^\circ$, the axes are oriented as shown in Figure 5. The final orientation of the actual axes with respect to the reference axes can be obtained with a sequence of four rotations. These are as follows:

1. Perform a positive rotation of E_2 about the YAW_2 axis to form an intermediate axis set X_2, Y_2, Z_2 .
2. Perform a positive rotation of E_2 about the X_2 axis to form a second intermediate axis set X_3, Y_3, Z_3 .
3. Perform a positive rotation of β_{24} about the Y_3 axis to form a third intermediate axis set X_4, Y_4, Z_4 .
4. Perform a negative rotation by δ_{A2} about the Z_4 axis (this corresponds to a positive rotation about the multisensor spin axis) to form the final orientation of the multisensor axes for $\theta_{R2} = -90^\circ$.

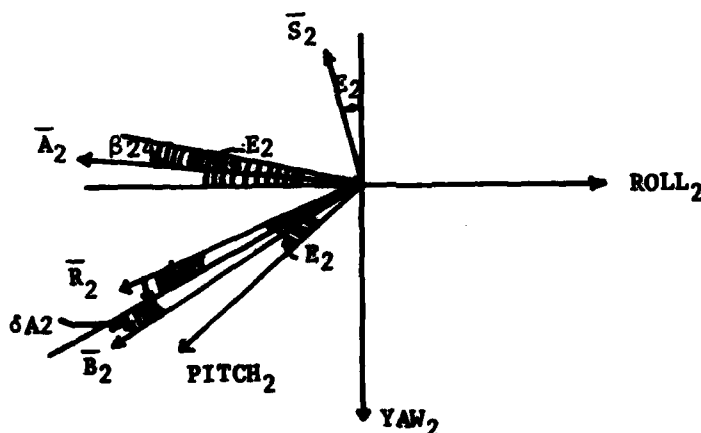


Figure 5. Multisensor 2 axes at $\theta_{R2} = -90^\circ$.

If one next forms the coordinate transformation matrices for each of these rotations, using the small angle approximation, the measurement equations for this orientation can be written as

$$A_{24} = -K_{A2} G_3 - E_2 G_2 + \delta_{A2} G_2 + \beta_{24} G_1 + B_{A2} \quad (9)$$

$$B_{24} = K_{A2} G_2 + \delta_{A2} G_3 - E_2 G_3 + E_2 G_1 + B_{B2}. \quad (10)$$

When multisensor 2 is rotated to $\theta_{R2} = -180^\circ$, the axes are oriented as shown in Figure 6. The final orientation of the actual axes with respect to the reference axis set can be obtained in this case with a sequence of three rotations. These are as follows:

1. Perform a positive rotation of $2E_2$ about the YAW_2 axis to form an intermediate axis set X_2, Y_2, Z_2 .
2. Perform a positive rotation of δ_{A2} about the X_2 axis to form a second intermediate axis set X_3, Y_3, Z_3 .
3. Perform a positive rotation of δ_{A2} about the X_3 axis (corresponding to a positive rotation of δ_{A2} about \bar{S}_2) to form the final orientation of the multisensor axes for $\theta_{R2} = -180^\circ$.

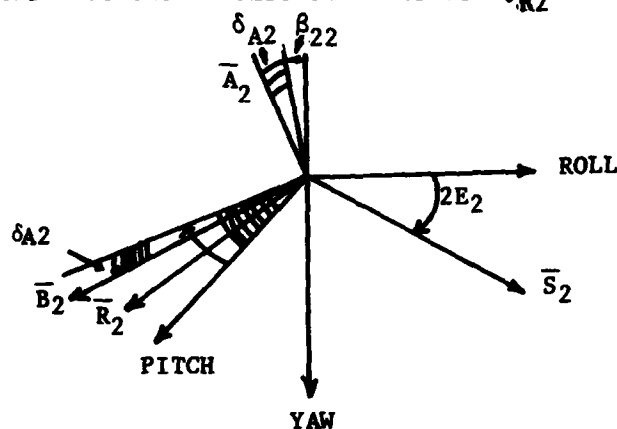


Figure 6. Multisensor 2 axes at $\theta_{R2} = -180^\circ$.

If one next forms the coordinate transformation matrices for each of these rotations, using the small angle approximation, the measurement equations for the $\theta_{R2} = -180^\circ$ orientation can be written as

$$A_{22} = -K_{A2}G_1 + \delta_{A2}G_2 - \beta_{22}G_3 + B_{A2} \quad (11)$$

$$B_{22} = K_{A2}G_2 - 2E_2G_3 + \delta_{A2}G_1 + B_{B2}. \quad (12)$$

The calibration equations for multisensor 2 are then developed by using the measurements at $\theta_{R2} = 0^\circ$ and -180° and the results are:

$$0.5 (A_{21} - A_{22} - \beta_{22}G_3) = K_{A2}G_1 \quad (13)$$

$$0.5 (B_{22} - B_{21}) + E_2G_3 = \delta_{A2}G_1 \quad (14)$$

$$0.5 (A_{21} + A_{22} + \beta_{22}G_3) - \delta_{A2}G_2 = B_{A2} \quad (15)$$

$$0.5 (B_{21} + B_{22}) + E_2G_3 - K_{A2}G_2 = B_{B2} \quad (16)$$

C. Multisensor 1 Accelerometer Calibration

Multisensor 1 is a roll/pitch sensor with the pitch axis a redundant axis. The sensor orientation during navigation is with $\theta_{R1} = -90^\circ$ as shown in Figure 1 and a reference axis set for use in calibration is as shown in Figure 3. Since the two multisensors are calibrated with respect to different reference axis sets, a transformation matrix relating the two reference sets is necessary. This transformation can be developed from a sequence of three small angle rotations as follows:

1. Perform a negative rotation of Ψ_Y about the YAW_1 axis to form an intermediate axis set X_2, Y_2, Z_2 .
2. Perform a negative rotation of Ψ_P about the Y_2 axis to form a second intermediate axis set X_3, Y_3, Z_3 .
3. Perform a negative rotation of Ψ_R about the X_3 axis to form the final configuration, which is the $ROLL_2, PITCH_2$, and YAW_2 axis set. The angles Ψ_Y, Ψ_P, Ψ_R represent small misalignment angles between the two reference axis sets and the transformation developed from the above sequence is represented by:

$$\begin{pmatrix} ROLL_2 \\ PITCH_2 \\ YAW_2 \end{pmatrix} = \begin{bmatrix} 1 & -\Psi_Y & \Psi_P \\ \Psi_Y & 1 & -\Psi_R \\ -\Psi_P & \Psi_R & 1 \end{bmatrix} \begin{pmatrix} ROLL_1 \\ PITCH_1 \\ YAW_1 \end{pmatrix} \quad (17)$$

This transformation is used to relate measurements made in one frame to the other frame during the calibration process.

Also, in accordance with the notation used in Reference 1, the gravitational components along the multisensor 1 reference axes are denoted by G_1^1, G_2^1, G_3^1 .

The accelerometer axes are assumed to be misaligned by δ_{A1} about the spin axis \bar{S}_1 , so the actual multisensor axes for $\theta_R = -90^\circ$ are located with respect to the reference axes as shown in Figure 7. The measurement equations for \bar{A}_1 and \bar{B}_1 , for this orientation, can be written as:

$$A_{14} = K_{A1} G_2^1 - \delta_{A1} G_3^1 + B_{A1} \quad (18)$$

$$B_{14} = K_{A1} G_3^1 + \delta_{A1} G_2^1 + B_{B1} \quad (19)$$

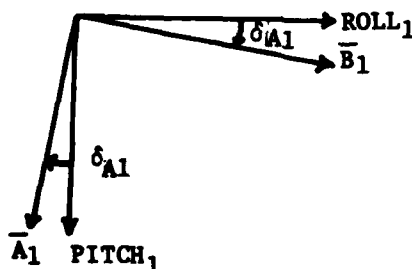


Figure 7. Accelerometer misalignment about the spin axis \bar{S}_1 , $\theta_{R1} = -90^\circ$.

When multisensor 1 is rotated to $\theta_{R1} = 0^\circ$, the axes are oriented as shown in Figure 8. The final orientation of the actual axes with respect to the reference axes can be obtained with a sequence of four small angle rotations as follows:

1. Perform a positive rotation of E_1 , about the $PITCH_1$ axis to form an intermediate axis set X_2, Y_2, Z_2 .
2. Perform a negative rotation of E_1 about the Z_2 axis to form a second intermediate axis set X_3, Y_3, Z_3 .
3. Perform a positive rotation of β_{11} about the X_3 axis to form a third intermediate axis set X_4, Y_4, Z_4 .
4. Perform a positive rotation of δ_{A1} about the Y_4 axis to form a final orientation of the multisensor axes for $\theta_{R1} = 0^\circ$.

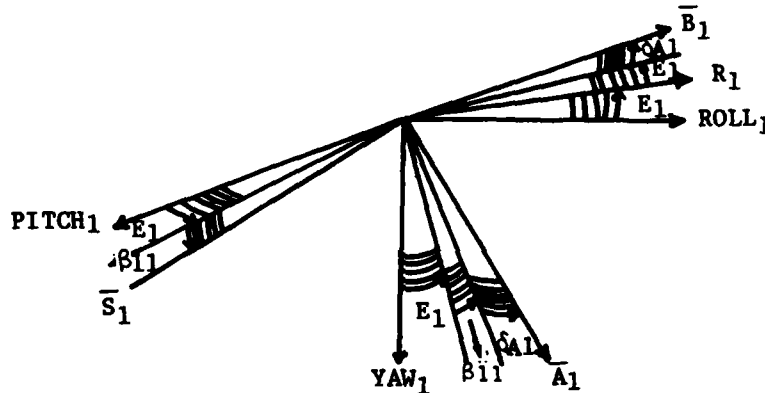


Figure 8. Multisensor 1 axes at $\theta_{R1} = 0^\circ$.

If one now forms the coordinate transformation matrix representing this rotation sequence, the measurement equations for $\theta_{R1} = 0^\circ$ may be expressed as:

$$A_{11} = K_{A1} G_1^1 + \delta_{A1} G_3^1 + E_1 G_3^1 - \beta_{11} G_2^1 + B_{A1} \quad (20)$$

$$B_{11} = K_{A1} G_3^1 - E_1 G_2^1 - E_1 G_1^1 - \delta_{A1} G_1^1 + B_{B1} \quad (21)$$

When multisensor 1 is rotated to $\theta_{R1} = -180^\circ$, the axes are oriented as shown in Figure 9. The final orientation of the actual axes with respect to the reference axis set can be obtained in this case with a sequence of four small angle rotations as follows:

1. Perform a positive rotation of E_1 about the $PITCH_1$ axis to form an intermediate axis set X_2, Y_2, Z_2 .

2. Perform a positive rotation of E_1 about the Z_2 axis to form a second intermediate axis set X_3, Y_3, Z_3 .
3. Perform a positive rotation of β_{12} about the X_3 axis to form a third intermediate axis set X_4, Y_4, Z_4 .
4. Perform a negative rotation of δ_{A1} about the Y_4 axis to obtain the final configuration for $\theta_{R1} = -180^\circ$.

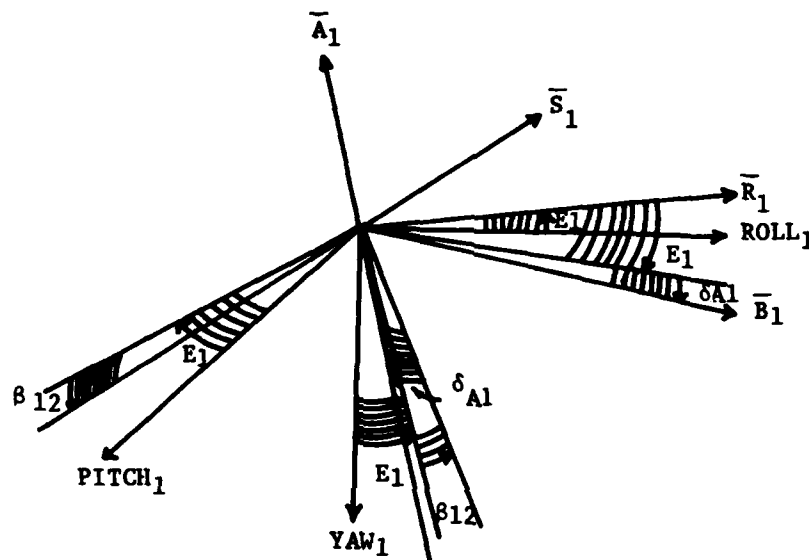


Figure 9. Multisensor 1 axes at $\theta_{R1} = -180^\circ$.

If one now forms the coordinate transformation matrix representing this rotation sequence, the measurement equations for $\theta_{R1} = -180^\circ$ may be expressed as:

$$A_{12} = -K_{A1} G_1^1 + \delta_{A1} G_3^1 - E_1 G_3^1 + \beta_{12} G_2^1 + B_{A1} \quad (22)$$

$$B_{12} = K_{A1} G_3^1 + E_1 G_2^1 + \delta_{A1} G_1^1 - E_1 G_1^1 + B_{B1} \quad (23)$$

The calibration equations for multisensor 1 are then developed by using the measurements at $\theta_{R1} = 0^\circ$ and -180° and the results are:

$$0.5 (A_{11} - A_{12} + (\beta_{11} + \beta_{12})G_2^1) = K_{A1} G_1^1 + E_1 G_3^1 \approx K_{A1} (G_1^1 + E_1 G_3^1) \quad (24)$$

$$0.5 (B_{12} - B_{11}) - E_1 G_2^1 = \delta A_1 G_1^1 \quad (25)$$

$$0.5 (A_{11} + A_{12} + (B_{11} - B_{12})G_2^1) - \delta A_1 G_3^1 = B_{A1} \quad (26)$$

$$0.5 (B_{11} + B_{12}) + E_1 G_1^1 - K_{A1} G_3^1 = B_{B1} \quad (27)$$

D. Iterative Calibration Procedure

As in Reference 1, the accelerometer parameters will be solved for by use of an iterative algorithm. The data necessary for this procedure are the accelerometer measurements taken at the stationary positions and the gravity components G_1 , G_2 , G_3 from a separate IMU. The iterative procedure for determining the accelerometer parameters for the A/W SHORADS case is shown in Table 2. Ten iterations are allowed in order that all cases have sufficient time to reach a steady-state solution.

TABLE 2. A/W SHORADS ACCELEROMETER ITERATIVE CALIBRATION PROCEDURE

PARAMETER	EQUATION
K_{A2}	13
δA_2	14
B_{B2}	16
K_{A1}	24
δA_1	25
B_{B1}	27
G_2	8
G_3^1	21
G_2^1	$G_2^1 = -\psi_Y G_3 + G_2 + \psi_R G_1$
G_3	$G_3 = G_3^1 - \psi_Y G_2^1 + \psi_P G_1^1$
B_{A2}	15
B_{A1}	26
G_1	$G_1 = \sqrt{G_0^2 - G_2^2 - G_3^2}$
G_1^1	$G_1^1 = \sqrt{G_0^2 - (G_2^1)^2 - (G_3^1)^2}$

IV. GYRO CALIBRATION

A. Introduction

This section develops the equations used in the gyro calibration process for the A/W SHORADS concept. The gyro calibration process utilizes

measurements made at the stationary positions to calibrate the gyro drift parameters. The gyro scale factor K_G and the gyro misalignment about the spin axis, δ_G , are calibrated using integrated angular rate measurements made as the multisensors are rotated. The reference axes defined for each multisensor in Section III. A. are used in the gyro calibration. The error angles and misalignments considered are as defined in the accelerometer sections.

B. Multisensor 2 Gyro Calibration from Stationary Measurements

The multisensor 2 gyro static measurements as presented in this section will correspond to the same orientation sequence, and the appropriate figures, presented in Section III. B. for the accelerometer static measurements. The first orientation is for $\theta_{R2} = 0^\circ$ as shown in Figure 2. The components of earth rate, represented by $\Omega_1, \Omega_2, \Omega_3$, are measured along the $YAW_2, PITCH_2,$ and $ROLL_2$ reference axes, respectively. The values for $\Omega_1, \Omega_2,$ and Ω_3 are input from an independent IMU as are the gravity components.

The measurement equations for the $\theta_{R2} = 0^\circ$ orientation are as follows:

$$W_{A21} = -\Omega_1 + D_{A2} + G_2 D_{A2GSB} + G_1 D_{A2GSA} \quad (28)$$

$$W_{B21} = \Omega_2 + D_{B2} - G_1 D_{A2GSB} + G_2 D_{A2GSA} \quad (29)$$

The sign convention for the cross-axis sensitivity term, D_{A2GSB} , can be visualized by assuming a fictitious mass unbalance along the negative \bar{S}_2 axis as shown in Figure 10. From Figure 10 it can be seen that the drift about \bar{A}_2 due to a positive acceleration along \bar{B}_2 will be given by

$$\Delta W_{A21} = G_2 D_{A2GSB} \quad (30)$$

i.e., a positive acceleration along \bar{B}_2 along with a positive cross axis G-sensitive drift term, D_{A2GSB} , will result in a positive drift about \bar{A}_2 . The drift about \bar{B}_2 due to a positive acceleration along \bar{A}_2 is seen from Figure 10 to be

$$\Delta W_{B21} = -G_1 D_{A2GSB} \quad (31)$$

For the multisensor 2 orientation of $\theta_{R2} = -90^\circ$, the gyro measurement equations are

$$W_{A24} = -\Omega_3 + D_{A2} + G_2 D_{A2GSB} - G_3 D_{A2GSA} \quad (32)$$

$$W_{B24} = \Omega_2 + D_{B2} + G_3 D_{A2GSB} - G_2 D_{A2GSA} \quad (33)$$

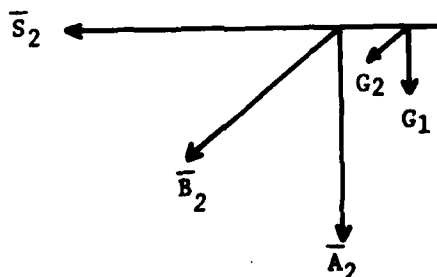


Figure 10. Multisensor 2 fictitious mass unbalance.

For the multisensor orientation of $\theta_{R2} = -180^\circ$ the gyro measurement equations are

$$W_{A22} = \Omega_1 + D_{A2} + G_2 D_{A2GSB} - G_1 D_{A2GSA} \quad (34)$$

$$W_{B22} = \Omega_2 + D_{B2} + G_1 D_{A2GSB} + G_2 D_{A2GSA} \quad (35)$$

The G-sensitive drift terms for multisensor 2 are calibrated by using the measurements at $\theta_{R2} = 0^\circ$ and $\theta_{R2} = -180^\circ$. The equations are as follows:

$$D_{A2GSA} = 0.5 (W_{A21} - W_{A22} + 2\Omega_1)/G_1 \quad (36)$$

$$D_{A2GSB} = 0.5 (W_{B22} - W_{B21})/G_1 \quad (37)$$

The gyro bias terms, D_{A2} and D_{B2} are calibrated from the measurements at $\theta_{R2} = 0^\circ$. The equations are as follows:

$$D_{A2} = 0.5 (W_{A21} + W_{A22}) - D_{A2GSB} G_2 \quad (38)$$

$$D_{B2} = 0.5 (W_{B22} + W_{B21}) - \Omega_2 - D_{A2GSA} G_2 . \quad (39)$$

C. Multisensor 1 Gyro Calibration from Stationary Measurements

The multisensor 1 gyro static measurements as presented in this section will correspond to the same orientation sequence, and the appropriate figures, presented in Section III. C. for the accelerometer static measurements. The first orientation is for $\theta_{R1} = -90^\circ$ as shown in Figure 3. The components of earth rate, represented by $\Omega_1^1, \Omega_2^1, \Omega_3^1$, are measured along the YAW₁, PITCH₁ and ROLL₁ axes, respectively. These earth rate components are related to those along the multisensor 2 reference axes by the transformation matrix given in equation 17.

The measurement equations for the $\theta_{R1} = -90^\circ$ orientation are:

$$W_{A14} = \Omega_2^1 + D_{A1} + G_3^1 D_{A1GSB} + G_2^1 D_{A1GSA} \quad (40)$$

$$W_{B14} = \Omega_3^1 + D_{B1} - G_2^1 D_{A1GSB} + G_3^1 D_{A1GSA} . \quad (41)$$

For the multisensor 1 orientation of $\theta_{R1} = 0^\circ$, the gyro measurement equations are

$$W_{A11} = -\Omega_1^1 + D_{A1} + G_3^1 D_{A1GSB} + G_1^1 D_{A1GSA} \quad (42)$$

$$W_{B11} = \Omega_3^1 + D_{B1} - G_1^1 D_{A1GSB} + G_3^1 D_{A1GSA} . \quad (43)$$

For the multisensor 1 orientation of $\theta_{R1} = -180^\circ$, the gyro measurement equations are

$$W_{A12} = \Omega_1^1 + D_{A1} + G_3^1 D_{A1GSB} - G_1^1 D_{A1GSA} \quad (44)$$

$$W_{B12} = \Omega_3^1 + D_{B1} + G_1^1 D_{A1GSB} + G_3^1 D_{A1GSA} . \quad (45)$$

The G-sensitive drift terms for multisensor 1 are calibrated by using the measurements at $\theta_{R1} = 0^\circ$ and -180° . The equations are:

$$D_{A1GSA} = 0.5(W_{A11} - W_{A12} + 2\Omega_1^1)/G_1^1 \quad (46)$$

$$D_{A1GSB} = 0.5(W_{B11} - W_{B12})/G_1^1 \quad (47)$$

The gyro bias terms, D_{A1} and D_{B1} are calibrated from the measurements at $\theta_{R1} = -90^\circ$. The equations are:

$$D_{A1} = 0.5(W_{A11} + W_{A12}) - G_3^1 D_{A1GSB} \quad (48)$$

$$D_{B1} = 0.5(W_{B11} + W_{B12}) - \Omega_3^1 - G_3^1 D_{A1GSA} \quad (49)$$

D. Multisensor 2 Gyro Scale Factor and Misalignment about Spin Axis Calibration

The calibration of K_{G2} and δ_{G2} utilizes angular rate measurements that are integrated into angular displacement measurements as the multisensor is rotated from $\theta_{R2} = 0^\circ$ to $\theta_{R2} = -180^\circ + \beta_{22}$. The angular rate measurement equations during rotation are:

$$W_{A2} = K_{G2}\Omega_{A2} + \delta_{G2}(\dot{\theta}_{R2} + \Omega_2) + D_{A2} + D_{A2GSA}A_2 + D_{A2GSB}B_2 \quad (50)$$

$$W_{B2} = K_{G2}(\dot{\theta}_{R2} + \Omega_2) - \delta_{G2}\Omega_{A2} + D_{B2} - D_{A2GSB}A_2 + D_{A2GSA}B_2 \quad (51)$$

$$\text{where } \Omega_{A2} = -\Omega_1 \cos \theta_{R2} + \Omega_3 \sin \theta_{R2} \quad (52)$$

The measured angular rates are then integrated over the rotation interval $t_{F2} - t_{O2} = t_{R2}$ to arrive at the following angular displacement measurements:

$$\theta_{A2} = \int_{t_{O2}}^{t_{F2}} W_{A2} dt = K_{G2} \theta_{\Omega A2} + \delta_{G2} (\Delta \theta_{R2} + \Omega_2 t_{R2}) + D_{A2} t_{R2} + D_{A2GSA} \Delta V_{A2} + D_{A2GSB} \Delta V_{B2} \quad (53)$$

$$\theta_{B2} = \int_{t_{O2}}^{t_{F2}} W_{B2} dt = K_{G2} (\Delta \theta_{R2} + \Omega_2 t_{R2}) - \delta_{G2} \theta_{\Omega A2} + D_{B2} t_{R2} + D_{A2GSA} \Delta V_{B2} - D_{A2GSB} \Delta V_{A2} \quad (54)$$

where

$$\theta_{\Omega A2} = \int_{t_{O2}}^{t_{F2}} \Omega_{A2} dt \quad (55)$$

$$\Delta \theta_{R2} = \int_{t_{O2}}^{t_{F2}} \dot{\theta}_{R2} dt = (-180^\circ + \beta_{22}) \quad (56)$$

$$\Delta V_{A2} = \int_{t_{02}}^{t_{F2}} A_2 dt \quad (57)$$

$$\Delta V_{B2} = \int_{t_{02}}^{t_{F2}} B_2 dt \quad (58)$$

In order to integrate \bar{A}_2 and \bar{B}_2 general equations for the accelerometer measurements must be developed that will satisfy the three stationary orientations and the dynamic orientations that occur during rotation. The general equations for multisensor 2 are

$$A_2 = K_{A2} (G_1 \cos(\dot{\theta}_{R2}t) + G_3 \sin(\dot{\theta}_{R2}t)) + \delta_{A2} G_2 + B_{A2} + E_2 (G_2 \sin(\dot{\theta}_{R2}t)) \quad (59)$$

$$B_2 = K_{A2} G_2 - \delta_{A2} (G_1 \cos(\dot{\theta}_{R2}t) + G_3 \sin(\dot{\theta}_{R2}t)) + B_{B2} - E_2 (G_1 \sin(\dot{\theta}_{R2}t))$$

$$-E_2 G_3 (1 - \cos(\dot{\theta}_{R2}t)). \quad (60)$$

These correspond to measurements along the \bar{A}_2 and \bar{B}_2 multisensor measurement axes, respectively.

After the six second rotation and simultaneous integration occur, all the terms in equations (53) and (54) are known from measurements or calculation except for δ_{G2} and K_{G2} . The gyro drift parameters were calibrated from stationary measurements. The error angle β_{22} is available from laboratory calibration. The earth rates are computed inputs from the IMU module. Thus, equations (53) and (54) can be used to calibrate δ_{G2} and K_{G2} .

Matrix inversion is chosen as the most effective method to solve for δ_{G2} and K_{G2} . Equations (53) and (54) are rewritten as

$$\theta_A = \theta_{A2} - D_{A2} t_{R2} - D_{A2GSA} \Delta V_{A2} - D_{A2GSB} \Delta V_{B2} = K_{G2} \theta_{\Omega A2} + \delta_{G2} (\Delta \theta_{R2} + \Omega_2 t_{R2}) \quad (61)$$

$$\theta_B = \theta_{B2} - D_{B2} t_{R2} + D_{A2GSB} \Delta V_{A2} - D_{A2GSA} \Delta V_{B2} = K_{G2} (\Delta \theta_{R2} + \Omega_2 t_{R2}) - \delta_{G2} \theta_{\Omega A2} \quad (62)$$

and can be expressed in matrix form as

$$\begin{pmatrix} \theta_A \\ \theta_B \end{pmatrix} = \begin{bmatrix} \theta_{\Omega A2} & \Delta\theta_{R2} + \Omega_2 t_{R2} \\ \Delta\theta_{R2} + \Omega_2 t_{R2} & -\theta_{\Omega A2} \end{bmatrix} \begin{pmatrix} K_{G2} \\ \delta_{G2} \end{pmatrix} \quad (63)$$

After the matrix inversion is performed, one obtains

$$\begin{pmatrix} K_{G2} \\ \delta_{G2} \end{pmatrix} = \frac{1}{\text{DETERMINANT}} \begin{bmatrix} -\theta_{\Omega A2} & -(\Delta\theta_{R2} + \Omega_2 t_{R2}) \\ -(\Delta\theta_{R2} + \Omega_2 t_{R2}) & \theta_{\Omega A2} \end{bmatrix} \begin{pmatrix} \theta_A \\ \theta_B \end{pmatrix} \quad (64)$$

where

$$\text{Determinant} = -((\theta_{\Omega A2})^2 + (\Delta\theta_{R2} + \Omega_2 t_{R2})^2). \quad (65)$$

One can now solve for K_{G2} and δ_{G2} as follows:

$$K_{G2} = ((\theta_{A2} - D_{A2} t_{R2} - D_{A2} GSA^{\Delta V} A2 - D_{A2} GSB^{\Delta V} B2) \theta_{\Omega A2} + (\theta_{B2} - D_{B2} t_{R2} + D_{A2} GSB^{\Delta V} A2 - D_{A2} GSA^{\Delta V} B2) (\Delta\theta_{R2} + \Omega_2 t_{R2})) / ((\theta_{\Omega A2})^2 + (\Delta\theta_{R2} + \Omega_2 t_{R2})^2) \quad (66)$$

$$\delta_{G2} = ((\theta_{A2} - D_{A2} t_{R2} - D_{A2} GSA^{\Delta V} A2 - D_{A2} GSB^{\Delta V} B2) (\Delta\theta_{R2} + \Omega_2 t_{R2}) - (\theta_{B2} - D_{B2} t_{R2} + D_{A2} GSB^{\Delta V} A2 - D_{A2} GSA^{\Delta V} B2) \theta_{\Omega A2}) / ((\theta_{\Omega A2})^2 + (\Delta\theta_{R2} + \Omega_2 t_{R2})^2) \quad (67)$$

E. Multisensor 1 Gyro Scale Factor and Misalignment about Spin Axis Calibration

The calibration of K_{G1} and δ_{G1} utilizes angular rate measurements that are integrated into angular displacement measurements as the multisensor is rotated from $\theta_{R1} = \beta_{11}$ to $\theta_{R1} = -180^\circ + \beta_{12}$. The angular rate measurement equations during rotation are:

$$W_{A1} = K_{G1} \Omega_{A1} + \delta_{G1} (\dot{\theta}_{R1} + \Omega_3^1) + D_{A1} + D_{A1} GSA A_1 + D_{A1} GSB B_1 \quad (68)$$

$$W_{B1} = K_{G1} (\dot{\theta}_{R1} + \Omega_3^1) - \delta_{G1} \Omega_{A1} + D_{B1} - D_{A1} GSB A_1 + D_{A1} GSA B_1, \quad (69)$$

$$\text{where } \Omega_{A1} = -\Omega_1^1 \cos \theta_{R1} - \Omega_2^1 \sin \theta_{R1}. \quad (70)$$

The measured angular rates are then integrated over the rotation interval $t_{F1} - t_{01} = t_{R1}$ to arrive at the following angular displacement measurements:

$$\theta_{A1} = \int_{t_{01}}^{t_{F1}} \omega_{A1} dt = K_{G1} \theta_{\Omega A1} + \delta_{G1} (\Delta \theta_{R1} + \Omega_3^1 t_{R1}) + D_{A1} t_{R1} + D_{A1GSA} \Delta v_{A1} + D_{A1GSB} \Delta v_{B1} \quad (71)$$

$$\theta_{B1} = \int_{t_{01}}^{t_{F1}} \omega_{B1} dt = K_{G1} (\Delta \theta_{R1} + \Omega_3^1 t_{R1}) - \delta_{G1} \theta_{\Omega A1} + D_{B1} t_{R1} + D_{A1GSA} \Delta v_{B1} - D_{A1GSB} \Delta v_{A1} \quad (72)$$

where

$$\theta_{\Omega A1} = \int_{t_{01}}^{t_{F1}} \Omega_{A1} dt \quad (73)$$

$$\Delta \theta_{R1} = \int_{t_{01}}^{t_{F1}} \dot{\theta}_{R1} dt = (-180^\circ - \beta_{11} + \beta_{12}) \quad (74)$$

$$\Delta v_{A1} = \int_{t_{01}}^{t_{F1}} A_1 dt \quad (75)$$

$$\Delta v_{B1} = \int_{t_{01}}^{t_{F1}} B_1 dt \quad (76)$$

In order to integrate A_1 and B_1 , general equations for the accelerometer measurements must be developed that will satisfy conditions at the three stationary orientations as well as providing data during the -180° rotation. The general equations for multisensor 1 are

$$\begin{aligned} A_1 = & K_{G1} (G_1^1 \cos(\dot{\theta}_{R1} t) - G_2^1 \sin(\dot{\theta}_{R1} t)) + \delta_{A1} G_3^1 (1 + 2 \sin(\dot{\theta}_{R1} t)) \\ & + B_{A1} + E_1 (G_3^1 \cos(\dot{\theta}_{R1} t)) \end{aligned} \quad (77)$$

$$B_1 = K_{A1} G_3^1 - \delta_{A1} (G_1^1 \cos(\dot{\theta}_{R1}t) + G_2^1 \sin(\dot{\theta}_{R1}t)) + B_{B1} - E_1 (G_2^1 \cos(\dot{\theta}_{R1}t)) - E_1 G_1^1 (1 + \sin(\dot{\theta}_{R1}t)) . \quad (78)$$

These correspond to measurements along the \bar{A}_1 and \bar{B}_1 multisensor measurement axes, respectively.

After the six second rotation and simultaneous integration occur all the terms in equations (71) and (72) are known from measurements or calculation except for δ_{G1} and K_{G1} . The gyro drift parameters were calibrated from stationary measurements. The error angles β_{11} and β_{12} are available from laboratory calibration. The earth rates are computed inputs from the IMU module. Thus, equations (71) and (72) can be used to calibrate δ_{G1} and K_{G1} .

As in Section IV. D., matrix inversion is used to solve for δ_{G1} and K_{G1} . Equations (71) and (72) are rewritten as:

$$\theta_A = \theta_{A1} - D_{A1} t_{R1} - D_{A1} GSA^{\Delta V} A1 - D_{A1} GSB^{\Delta V} B1 = K_{G1} \theta_{\Omega A1} + \delta_{G1} (\Delta \theta_{R1} + \Omega_3^1 t_{R1}) \quad (79)$$

$$\theta_B = \theta_{B1} - D_{B1} t_{R1} + D_{A1} GSB^{\Delta V} A1 - D_{A1} GSA^{\Delta V} B1 = K_{G1} (\Delta \theta_{R1} + \Omega_3^1 t_{R1}) - \delta_{G1} \theta_{\Omega A1} , \quad (80)$$

which, in matrix form, is expressible as:

$$\begin{pmatrix} \theta_A \\ \theta_B \end{pmatrix} = \begin{bmatrix} \theta_{\Omega A1} & (\Delta \theta_{R1} + \Omega_3^1 t_{R1}) \\ (\Delta \theta_{R1} + \Omega_3^1 t_{R1}) & -\theta_{\Omega A1} \end{bmatrix} \begin{pmatrix} K_{G1} \\ \delta_{G1} \end{pmatrix} . \quad (81)$$

After the matrix inversion is performed, one obtains:

$$\begin{pmatrix} K_{G1} \\ \delta_{G1} \end{pmatrix} = \frac{1}{\text{DETERMINANT}} \begin{bmatrix} -\theta_{\Omega A1} & -(\Delta \theta_{R1} + \Omega_3^1 t_{R1}) \\ -(\Delta \theta_{R1} + \Omega_3^1 t_{R1}) & \theta_{\Omega A1} \end{bmatrix} \begin{pmatrix} \theta_A \\ \theta_B \end{pmatrix} , \quad (82)$$

where

$$\text{DETERMINANT} = -(\theta_{\Omega A1})^2 + (\Delta\theta_{R1} + \Omega_3^1 t_{R1})^2. \quad (83)$$

One can thus solve for K_{G1} and δ_{G1} as follows:

$$K_{G1} = ((\theta_{A1} - D_{A1} t_{R1} - D_{A1GSA} \Delta V_{A1} - D_{A1GSB} \Delta V_{B1}) \theta_{\Omega A1} + (\theta_{B1} - D_{B1} t_{R1} + D_{A1GSB} \Delta V_{A1} - D_{A1GSA} \Delta V_{B1}) (\Delta\theta_{R1} + \Omega_3^1 t_{R1})) / ((\theta_{\Omega A1})^2 + (\Delta\theta_{R1} + \Omega_3^1 t_{R1})^2) \quad (84)$$

$$\delta_{G1} = ((\theta_{A1} - D_{A1} t_{R1} - D_{A1GSA} \Delta V_{A1} - D_{A1GSB} \Delta V_{B1}) (\Delta\theta_{R1} + \Omega_3^1 t_{R1}) - (\theta_{B1} - D_{B1} t_{R1} + D_{A1GSB} \Delta V_{A1} - D_{A1GSA} \Delta V_{B1}) \theta_{\Omega A1}) / ((\theta_{\Omega A1})^2 + (\Delta\theta_{R1} + \Omega_3^1 t_{R1})^2). \quad (85)$$

REFERENCE

1. "Low Cost Multifunction Sensor Phase I Technical Report," Rockwell International, Collins Government Avionics Division Report, Contract DAAB01-82-C-A309, Cedar Rapids, Iowa, November 12, 1982.

APPENDIX. CALIBRATION PROCEDURE

The calibration procedure described in this report was programmed into an alignment subroutine in the A/W SHORADS 6-DOF digital simulation. The nomenclature used in the subroutine is equated to the nomenclature used in this report in the program dictionary given in Table A-1, a listing of the calibration algorithm is given in Table A-2. The results of several check runs are presented in Table A-3.

The procedure for calibrating the gyro and accelerometer parameters was simulated in the following sequence:

(1) Known parameters are set (laboratory calibrations permit knowledge of error angles and misalignment angles and literature from the manufacturer of the multisensor provides additional information necessary).

(2) The IMU provides measurements of earth and gravity rates.

(3) Static measurements are made at 0° , -90° , -180° .

(4) Angular rate measurements are made as the multisensor rotates from 0° to -180° , and these measurements are integrated into angular displacements (the total time necessary for the rotation is six seconds; the rate of rotation is -30° per second).

(5) Ten iterations of the accelerometer calibration equation set are made in order to provide sufficient time for any extreme errors to settle out to good solutions. From this iterative procedure all the accelerometer parameters are calibrated.

(6) The gyro drift parameters are calibrated from static measurements made by the multisensor and the estimated gravity terms from step 5.

(7) All terms in equations (53) and (54) for multisensor 2 and equations (71) and (72) for multisensor 1 are known from measurements or calculation except for δ_{G1} and K_{G1} . Thus, these equations are used to calibrate δ_{G1} and K_{G1} .

(8) Calibrated error terms for both accelerometer and gyro models are computed.

TABLE A-1. CALIBRATION PROGRAM DICTIONARY

FORTTRAN NAME	VARIABLE	DEFINITION
A_{ij} ($i=1,2; j=1,2,4$)		Measured A_j axis acceleration measurement at orientation j ($j=1$ for $\theta_{R1} = 0^\circ$, $j=2$ for $\theta_{R1} = -180^\circ$, $j=4$ for $\theta_{R1} = -90^\circ$)
ABO	G_0	Magnitude of earth gravity at present position.
AB1, AB2, AB3	G_1, G_2, G_3	Components of gravity (nominally along roll, pitch, and yaw) corresponding to the ideal coordinate frame defined for multisensor 2.
AB1PRM, AB2PRM AB3PRM	G_1^1, G_2^1, G_3^1	Components of gravity corresponding to the ideal coordinate frame defined for multisensor 1.
ABIASX ABIASY ABIASZ		Accelerometer bias along ACS X-axis Accelerometer bias along ACS Y-axis Accelerometer bias along ACS Z-axis
AC1 AC2 AL1 AL2		Pitch-yaw cross G-sensitivity Roll-pitch cross G-sensitivity Pitch-yaw inline G-sensitivity Roll-pitch inline G-sensitivity
ALNFLG		Align control flag
AML1, AML2	$\delta A_1, \delta A_2$	Accelerometer misalignments about the spin axes.
ASF1, ASF2	K_{A1}, K_{A2}	Accelerometer scale factor
ASFX ASFY ASFZ		Accelerometer scale factor error along ACS X-axis. Accelerometer scale factor error along ACS Y-axis. Accelerometer scale factor error along ACS Z-axis.
B_{ij} ($i=1,2; j=1,2,4$)		Measured B_j axis acceleration measurement at orientation j .
BA11-BA33		Elements of BCS to ACS transformation matrix.

TABLE A-1. CALIBRATION PROGRAM DICTIONARY (CONTINUED)

FORTTRAN NAME	VARIABLE	DEFINITION
BA1, BA2 BB1, BB2	BA1, BA2 BB1, BB2	Accelerometer biases for axes A ₁ . Accelerometer biases for axes B ₁ .
BG11-BG33		Elements of BCS to GCS transformation matrix.
BTA11, BTA12	β_{11}, β_{12}	Rotation angle errors for multisensor 1.
BTA21, BTA22	β_{21}, β_{22}	Rotation angle errors for multisensor 2.
CTHR1, CTHR2	$\Delta\theta_{R1}, \Delta\theta_{R2}$	Total mechanical rotation angle.
DA1, DA2 DB1, DB2	DA1, DA2 DB1, DB2	Gyro drift bias for axes A ₁ and A ₂ . Gyro drift bias for axes B ₁ and B ₂ .
DA1GSA, DA2GSA	DA1GSA, DA2GSA	Gyro G-sensitive drift sensitivity to acceleration along the angular rate axis.
DA1GSB, DA2GSB	DA1GSB, DA2GSB	Gyro G-sensitive drift cross-axis.
DCMPPT	ψ_P	Misalignment angle in pitch.
DCMPRL	ψ_R	Misalignment angle in roll.
DCMPYA	ψ_Y	Misalignment angle in yaw.
DGSA1, DGSA2	A ₁ , A ₂	Multisensor measurements along the A ₁ axis.
DBSB1, DGSB2	B ₁ , B ₂	Multisensor measurements along the B ₁ axis.
DOME1, DOME2	Ω_{A1}, Ω_{A2}	Projection of earth rate onto the A ₁ axes.
E1, E2	E ₁ , E ₂	Non-orthogonality of rotation axis to spin axis.
EAA	ϵ_{QA}	Misalignment of the pitch-yaw multisensor due to temperature variations.
EAAP	ϵ'_{QA}	Misalignment of the roll-pitch multisensor due to temperature variations.

TABLE A-1. CALIBRATION PROGRAM DICTIONARY (CONTINUED)

FORTTRAN NAME	VARIABLE	DEFINITION
EAW1		Separation of ACS and GCS Y-axis due to temperature variations.
EAW2		Separation of ACS and GCS Y-axis due to temperature variations.
EAW3		Separation of ACS and GCS Z-axis due to temperature variations.
EB1	ϵ_{B1}	Misalignment in azimuth between pitch-yaw multisensor spin axis and BCS X-axis.
EB2	ϵ_{B2}	Misalignment in elevation between pitch-yaw multisensor spin axis and BCS Y-axis.
EOA	ϵ_{OA}	Orthogonality error in the pitch-yaw multisensor normal to the spin axis.
EOAP	ϵ'_{OA}	Orthogonality error in the roll-pitch multisensor normal to the spin axis.
ETAXAC	η_{xBCS}	Component of gravity along BCS X-axis.
ETAYAC	η_{yBCS}	Component of gravity along BCS Y-axis.
ETAZAC	η_{zBCS}	Component of gravity along BCS Z-axis.
GBIASP		Gyro roll rate bias.
GBIASQ		Gyro pitch rate bias.
GBIASR		Gyro yaw rate bias.
GL1		Accelerometer roll rate sensitivity.
GL2		Accelerometer pitch rate sensitivity.
GML1, GML2	δ_{G1}, δ_{G2}	Gyro misalignments about the spin axis.
GSA1, GSA2	$\Delta v_{A1}, \Delta v_{A2}$	Value of integrated acceleration over interval τ_{R1} .
GSB1, GSB2	$\Delta v_{B1}, \Delta v_{B2}$	Value of integrated acceleration over interval τ_{R1} .
GSF1, GSF2	K_{G1}, K_{G2}	Gyro scale factor.

TABLE A-1. CALIBRATION PROGRAM DICTIONARY (CONTINUED)

FORTTRAN NAME	VARIABLE	DEFINITION
GSFP GSFQ GSFR		Gyro roll rate scale factor error. Gyro pitch rate scale factor error. Gyro yaw rate scale factor error.
LAST		Flag that controls six second integration.
LST		Flag that controls parameter initialization.
OME1, OME2	$\theta_{\Omega A1}, \theta_{\Omega A2}$	Value of integral of Ω_{A1} obtained over interval tr_1 .
P	P	Missile roll rate.
PHII	ϕ_1	Misalignment in attitude between multisensor and BCS frame.
PSI12, PSI13, PSI21 PSI23, PSI31, PSI32		Elements of BCS to GCS transformation matrix (compensate for non-orthogonality and misalignment errors).
PSII	ψ_1	Misalignment in azimuth between multisensor and BCS frame.
Q	Q	Missile pitch rate.
QDT		Quantization time rate.
R	R	Missile yaw rate.
RATE	$\dot{\theta}_{R1}$	Mechanical rotation rate.
R1, R2, R3	$\Omega_1, \Omega_2, \Omega_3$	Components of earth rate (nominally along roll, pitch, yaw) corresponding to the ideal coordinate frame defined for multisensor 2.
R1PRM, R2PRM, R3PRM	$\Omega_1^1, \Omega_2^1, \Omega_3^1$	Components of earth rate corresponding to the ideal coordinate frame defined for multisensor 1.
RD	θ_{R1}	Angular orientation of rotation mechanisms.
$\overline{\text{Roll}}^2, \overline{\text{Pitch}}^2, \overline{\text{Yaw}}^2$		Ideal coordinate frame defined for multisensor 2.

TABLE A-1. CALIBRATION PROGRAM DICTIONARY (CONTINUED)

FORTRAN NAME	VARIABLE	DEFINITION
Roll ¹ , Pitch ¹ , Yaw ¹		Ideal coordinate frame defined for multisensor 1.
SDVX		Gravity component along the ACS X-axis.
SDVY		Gravity component along the ACS Y-axis.
SDVZ		Gravity component along the ACS Z-axis.
TA1, TA2	θ_{A1}, θ_{A2}	Value of integrated angular rates over intervals t_{R1} .
TB1, TB2	θ_{B1}, θ_{B2}	Value of integrated angular rates over intervals t_{R1} .
TALIGN	t_{O1}, t_{O2}	Lower limit of integration during rotation.
TF1, TF2	t_{F1}, t_{F2}	Upper limit of integration during rotation.
THT12, THT13, THT21 THT23, THT31, THT32		Elements of BCS to ACS transformation matrix (compensate for non-orthogonality and misalignment errors).
THTAI	θ_1	Misalignment in elevation between multisensor and BCS frame.
TIME		Simulation time.
TR1, TR2	t_{R1}, t_{R2}	Times to rotate from 0° to -180° orientation.
WA _{1j} (i=1,2;j=1,2,4)		Measured A _i axis angular rates at orientation j.
WB _{1j} (i=1,2;j=1,2,4)		Measured B _i axis angular rates at orientation j.

TABLE A-2. CALIBRATION ALGORITHM

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PROGRAM LISTING
IF(LST.EQ.1)GO TO 77
IF(SNGL(TIME).LT.(TALION-QDT+0.0001)) GO TO 5050
IF(ALNFLQ.GE.0.5)GO TO 5045
WRITE(6,99) SNGL(TIME)
99 FORMAT(2X,'ROCKWELL-COLINS CAL. CALLED AT ',F11.4,' SEC.')
C*****
C***** SET PARAMETERS VIA MCARLD VALUES
C*****
ASF1 = ASFY + 1.0
ASF2 = ASFZ + 1.0
BA1 = ABIASY
BB1 = ABIASX
BA2 = ABIASZ
BB2 = ABIASY
AML1 = EAAP
AML2 = EAA
QSF1 = QSFQ
QSF2 = QSFR
DB1 = QBIASP
DA1 = QBIASQ
DB2 = QBIASQ
DA2 = QBIASR
QML1 = EAW1
QML2 = EAW2
DA20SA = AL1
DA20SB = AC1
DA10SA = AL2
DA10SB = AC2

C
C THE ACTUAL GRAVITY AND EARTH RATE COMPONENTS
C ACTING, RESPECTIVELY, ALONG AND ABOUT THE MULTISENSOR
C SENSITIVE AXES ARE AS FOLLOWS:
C
AB3 = BAI11*ETAX00 + BAI12*ETAY00 + BAI13*ETAZ00
AB2 = BAI21*ETAX00 + BAI22*ETAY00 + BAI23*ETAZ00
AB1 = BAI31*ETAX00 + BAI32*ETAY00 + BAI33*ETAZ00
AB0 = SQRT(AB1*AB1+AB2*AB2+AB3*AB3)
AB1PRH = +DCMPPT*AB3 - DCMPL*AB2 + AB1
AB2PRH = -DCMPYA*AB3 + AB2 + DCMPL*AB1
AB3PRH = AB3 + DCMPYA*AB2 - DCMPT*AB1
R3 = BAI11*OMEQXB + BAI12*OMEQYB + BAI13*OMEQZB
R2 = BAI21*OMEQXB + BAI22*OMEQYB + BAI23*OMEQZB
R1 = BAI31*OMEQXB + BAI32*OMEQYB + BAI33*OMEQZB
R1PRH = +DCMPPT*R3 - DCMPL*R2 + R1
R2PRH = -DCMPYA*R3 + R2 + DCMPL*R1
R3PRH = R3 + DCMPYA*R2 - DCMPT*R1
LST = LST + 1

C
C ACCELEROMETER MODEL
C OUTPUTS MEASURED VALUES (INPUTS TO CALBRA' EGNS)
C
A21 = ASF2*AB1+AML2*AB2+BA2

```

```

A22 = -ASF2*AB1+AML2*AB2-BTA22*AB3+BA2
A24 = -ASF2*AB3-E2*AB2+AML2*AB2+BTA24*AB1+BA2
B21 = ASF2*AB2-AML2*AB1+BB2
B22 = ASF2*AB2-2.0*E2*AB3+AML2*AB1+BB2
B24 = ASF2*AB2+AML2*AB3-E2*AB3+E2*AB1+BB2
A11 = ASF1*AB1PRM+AML1*AB3PRM+E1*AB3PRM-BTA11*AB2PRM+BA1
A12 = -ASF1*AB1PRM+AML1*AB3PRM-E1*AB3PRM+BTA12*AB2PRM+BA1
A14 = ASF1*AB2PRM-AML1*AB3PRM+BA1
B11 = ASF1*AB3PRM-E1*AB2PRM-E1*AB1PRM-AML1*AB1PRM+BB1
B12 = ASF1*AB3PRM+E1*AB2PRM+AML1*AB1PRM-E1*AB1PRM+BB1
B14 = ASF1*AB3PRM+AML1*AB2PRM+BB1

```

C****

C***** GYRO MODEL

C****

```

WA21 = -R1+DA2+DA2GSB*AB2+DA2GSA*AB1
WB21 = R2+DB2-DA2GSB*AB1+DA2GSA*AB2
WA22 = R1+DA2+DA2GSB*AB2-DA2GSA*AB1
WB22 = R2+DB2+DA2GSB*AB1+DA2GSA*AB2
WA24 = -R3+DA2+DA2GSB*AB2-DA2GSA*AB3
WB24 = R2+DB2+DA2GSB*AB3+DA2GSA*AB2
WA14 = R2PRM+DA1+DA1GSB*AB3PRM+DA1GSA*AB2PRM
WB14 = R3PRM+DB1-DA1GSB*AB2PRM+DA1GSA*AB3PRM
WA11 = -R1PRM+DA1+DA1GSB*AB3PRM+DA1GSA*AB1PRM
WB11 = R3PRM+DB1-DA1GSB*AB1PRM+DA1GSA*AB3PRM
WA12 = R1PRM+DA1+DA1GSB*AB3PRM-DA1GSA*AB1PRM
WB12 = R3PRM+DB1+DA1GSB*AB1PRM+DA1GSA*AB3PRM

```

77 CONTINUE

IF(TIME.GT.7.000)GO TO 73

RATE = -30./DEGRAD

CTHR1 = -180./DEGRAD - BTA11 + BTA12

CTHR2 = -180./DEGRAD + BTA22

C

C

C

GENERAL EQUATIONS FOR INTEGRATION DURING ROTATION

```

TR2 = TIME
RD = RATE * TR2
DOME1 = -R1PRM * COS(RD) - R2PRM * SIN(RD)
DOME2 = -R1 * COS(RD) + R3 * SIN(RD)
DOSA1 = ASF1*(AB1PRM*COS(RD)-AB2PRM*SIN(RD))
+AML1*AB3PRM*(1.+2.*SIN(RD))+E1*(AB3PRM*COS(RD))+BA1
DGSB1 = ASF1*AB3PRM-AML1*(AB1PRM*COS(RD)+AB2PRM*SIN(RD))
+BB1-E1*(AB2PRM*COS(RD))-E1*AB1PRM*(1.+SIN(RD))
DOSA2 = ASF2*(AB1*COS(RD)+AB3*SIN(RD))+AML2*AB2+BA2
+E2*(AB2*SIN(RD))
DGSB2 = ASF2*AB2-AML2*(AB1*COS(RD)+AB3*SIN(RD))+BB2
-E2*(AB1*SIN(RD))-E2*AB3*(1.-COS(RD))
GO TO 74

```

73 CONTINUE

TA2=GSF2*DOME2+OHL2*(CTHR2+R2*TR2)+DA2*TR2+DA2GSA*GSA2

+DA2GSB*GSB2

TB2=GSF2*(CTHR2+R2*TR2)-OHL2*DOME2+DB2*TR2-DA2GSB*GSA2

+DA2GSA*GSB2

TA1=GSF1*DOME1+OHL1*(CTHR1+R3PRM*TR2)+DA1*TR2+DA1GSA*GSA1

```

+DA10SB*0SB1
TB1=0SF1*(CTHR1+R3PRM*TR2)-0HL1*0NE1+DB1*TR2-DA10SB*0SA1
+DA10SA*0SB1
C*****
C   THIS SECTION SIMULATES THE ROCKWELL-COLLINS
C   PRE-FLIGHT SENSOR CALIBRATIONS. IT IS ASSUMED
C   THAT ALL SPIN-UP, WAIT, AND ROTATIONAL TIMES
C   HAVE BEEN SATISFIED AND ACCELEROMETER MEASUREMENTS
C   ARE AVAILABLE. 10 ITERATIONS OF THE EQUATION
C   SET WILL BE USED TO MAKE SURE THAT EXTREME
C   ERRORS SETTLE OUT TO GOOD SOLUTIONS.
C*****
C
C
C   THE GRAVITY AND EARTH RATE COMPONENTS, AS MEASURED BY
C   AN EXTERNALLY LOCATED IMU, FOR USE IN THE CALIBRATION
C   COMPUTATIONS ARE AS FOLLOWS:

AB1 = 0ZEO
AB2 = 0YEO
AB3 = 0XEO
AB1PRM = +DCMPPT*AB3 - DCMPL*AB2 + AB1
AB2PRM = -DCMPYA*AB3 + AB2 + DCMPL*AB1
AB3PRM = AB3 + DCMPIA*AB2 - DCMPT*AB1
R1 = 0MEOZE
R2 = 0MEOYE
R3 = 0MEOXE
R1PRM = +DCMPPT*R3 - DCMPL*R2 + R1
R2PRM = -DCMPYA*R3 + R2 + DCMPL*R1
R3PRM = R3 + DCMPIA*R2 - DCMPT*R1
DO 110 I=1,10
ASF2 = (A21-A22-BTA22*AB3)/(2.0*AB1)
AML2 = (0.5*(B22-B21)+E2*AB3)/AB1
BB2 = .5*(B21+B22)+E2*AB3-ASF2*AB2
ASF1 = (A11-A12+(BTA11+BTA12)*AB2PRM)/(2.0*(AB1PRM+E1*AB3PRM))
AML1 = (0.5*(B12-B11)-E1*AB2PRM)/AB1PRM
BB1 = .5*(B11+B12)+E1*AB1PRM-ASF1*AB3PRM
AB2 = (B21+AML2*AB1-BB2)/ASF2
AB3PRM = (B11+E1*AB2PRM+E1*AB1PRM+AML1*AB1PRM-BB1)/ASF1
AB2PRM = -DCMPYA*AB3+AB2+DCMPL*AB1
AB3 = AB3PRM-DCMPYA*AB2PRM+DCMPT*AB1PRM
BA2 = .5*(A21+A22+BTA22*AB3)-AML2*AB2
BA1 = .5*(A11+A12+(BTA11+BTA12)*AB2PRM)-AML1*AB3PRM
AB1 = SQRT(AB0*AB0 - AB2*AB2 - AB3*AB3)
AB1PRM = SQRT(AB0*AB0 - AB2PRM*AB2PRM - AB3PRM*AB3PRM)
110 CONTINUE
DA20SA = (WA21-WA22+2.*R1)/(2.*AB1)
DA20SB = (WB22-WB21)/(2.*AB1)
DA2 = 0.5*(WA21+WA22)-DA20SB*AB2
DB2 = 0.5*(WB22+WB21)-R2-DA20SA*AB2
DA10SA = (WA11-WA12+2.*R1PRM)/(2.*AB1PRM)
DA10SB = (WB12-WB11)/(2.*AB1PRM)
DA1 = 0.5*(WA11+WA12)-DA10SB*AB3PRM
DB1 = 0.5*(WB11+WB12)-R3PRM-AB3PRM*DA10SA
0HL2=((CTHR2+R2*TR2)*(TA2-DA2*TR2-DA20SA*0SA2-DA20SB*0SB2)

```

```

<-ONE2*(TB2-DB2*TR2+DA20SB*OSA2-DA20SA*OSB2))
C/(((CTHR2+R2*TR2)**2+ONE2**2)
OSF2=(ONE2*(TA2-DA2*TR2-DA20SA*OSA2-DA20SB*OSB2)+(CTHR2
<+R2*TR2)*(TB2-DB2*TR2+DA20SB*OSA2-DA20SA*OSB2))
C/(((CTHR2+R2*TR2)**2+ONE2**2)
OSF1=((TA1-DA1*TR2-DA10SA*OSA1-DA10SB*OSB1)*ONE1+(TB1
<-DB1*TR2+DA10SB*OSA1-DA10SA*OSB1)*(CTHR1+R3PRM*TR2))
C/(((CTHR1+R3PRM*TR2)**2+ONE1**2)
OML1=((TA1-DA1*TR2-DA10SA*OSA1-DA10SB*OSB1)*(CTHR1
<+R3PRM*TR2)-ONE1*(TB1-DB1*TR2+DA10SB*OSA1-DA10SA*OSB1))
C/(((CTHR1+R3PRM*TR2)**2+ONE1**2)
PRINT*, 'CALIBRATED TERMS FROM ROTATION'
PRINT*, '*****'
PRINT*, 'OML2, OML1, OSF1, OSF2=', OML2, OML1, OSF1, OSF2
PRINT*, '*****'
SDVX = AB3
SDVY = AB2
SDVZ = AB1

```

```

C
C***** COMPUTE CALIBRATED ERROR TERMS FOR ACCEL. MODEL EQNS
C

```

```

C***** WRITE OUT MCARLO INPUT VALUES

```

```

WRITE(6,137)
WRITE(6,134) ASFX, ASFY, ASFZ
WRITE(6,135) ABIASX, ABIASY, ABIASZ
WRITE(6,136) EAA, EAAP
ASFY = ASFY-ASF1+1.0
ASFZ = ASFZ-ASF2+1.0
ASFX = ASFY
ABIASX = ABIASX-BB1
ABIASY = ABIASY-(BA1+BB2)/2.0
ABIASZ = ABIASZ-BA2
EAAP = EAAP-AML1
EAA = EAA-AML2
WRITE(6,138)
WRITE(6,134) ASFX, ASFY, ASFZ
WRITE(6,135) ABIASX, ABIASY, ABIASZ
WRITE(6,136) EAA, EAAP
THT13 = EAAP + EOPAP
THT31 = EAA + EOA
THT21 = EAA
BA11 = 1.0
BA12 = PSII + THT13
BA13 = -THTAI-THT12
BA21 = -PSII-THT23
BA22 = 1.0
BA23 = PHII+THT21
BA31 = THTAI+THT32
BA32 = -PHII-THT31
BA33 = 1.0
134 FORMAT(2X, 'ASFX, ASFY, ASFZ =', 3F12.6)
135 FORMAT(2X, 'ABIASX, ABIASY, ABIASZ =', 3F12.6)
136 FORMAT(2X, 'EAA, EAAP =', 2F12.6)
137 FORMAT(1H, 'MCARLO INPUT VALUES')
138 FORMAT(1H, 'CALIBRATED OUTPUT ERRORS')

```

```

C****
C***** COMPUTE CALIBRATED ERROR TERMS FOR GYRO MODEL
C****
  QSF0 = QSF0 - QSF1 + 1.0
  QSFR = QSFR - QSF2 + 1.0
  QSFP = QSF0
  QBIASP = QBIASP - DB1
  QBIASG = QBIASG - (DA1+DB2)/2.0
  QBIASR = QBIASR - DA2
  AL1 = AL1 - DA20SA
  AC1 = AC1 - DA20SB
  AL2 = AL2 - DA10SA
  AC2 = AC2 - DA10SB
  EAW1 = EAW1 - OML1
  EAW2 = EAW2 - OML2
  PSI13 = EAAP + EAW1
  PSI21 = EAA + EAW2
  B011 = 1.0
  B012 = PS11 + PSI13
  B013 = -THTAI - PS112
  B021 = -PS11 - PSI23
  B022 = 1.0
  B023 = PH11 + PSI21
  B031 = THTAI + PSI32
  B032 = -PH11 - PSI31
  B033 = 1.0
  STOP
  END

```

TABLE A-3. EXAMPLE RUNS

EXAMPLE RUN # 1

INPUT VALUES

[illegible]

37

```

0. 306612E+02 -. 456905E-01 -. 957834E+01
0. 306612E+02 -. 456905E-01 -. 957834E+01
0. 306612E+02 -. 456905E-01 -. 957834E+01
0. 306612E+02 -. 456905E-01 -. 957834E+01
0. 306612E+02 -. 456905E-01 -. 957834E+01
0. 306612E+02 -. 456905E-01 -. 957834E+01
0. 306612E+02 -. 456906E-01 -. 957834E+01
0. 306612E+02 -. 456906E-01 -. 957834E+01
0. 306612E+02 -. 456906E-01 -. 957834E+01
OUTPUT FROM GYRO STATIONARY MEASUREMENTS
      DA2GSA      DA2G5B      DA1GSA      DA1G5B
0. 145000E-02 0. 145000E-02 0. 145000E-02 0. 145000E-02
      DA2      DB2      DA1      DB1
0. 969959E-05 0. 970013E-05 0. 970252E-05 0. 970438E-05
      GML2      GML1      GSF2      GSF1
INPUT VALUES OF GYRO PARAMETERS
0. 500000E-06 0. 500000E-06 0. 500000E-02 0. 500000E-02
CALIBRATED TERMS FROM ROTATION
*****
      GML2      GML1      GSF2      GSF1
0. 488745E-06 0. 500521E-06 0. 500001E-02 0. 500000E-02
*****

```

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